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ON STATIC AND DYNAMIC COMPUTATIONS OF ONE-DIMENSIONAL REGULAR SYSTEMS

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Simplifications of the computations of statics and the small vibrations of regular mechanical structures are investigated. On the basis of the method of an elemenatry cell it is shown that these simplifications hold for all regular systems which are representable as elementary in the sense of some irreducible representation of the subgroup $D_{2h}^{(1)} \subset D_{2h}^{(1)*}$, where $D_{2h}^{(1)*}$ is the space symmetry group of the corresponding infinite regular system. The boundary conditions of such elementary systems are described in general form. The essence of the simplifications is the passage from a computation of the regular construction over to computations of a finite number of elementary systems in the sense of the group $D_{2h}^{(1)*}$ whose types are indicated. The loading of the elementary systems is defined by using a developed effective method of decomposing the load of the initial regular system.

A number of investigations [1 – 4] is devoted to a study of regular mechanical systems. These investigations are associated with translational symmetry of an infinite regular system in [2], which permitted use of the group representation theory apparatus developed for applications [5]. However, the most general and complete results in the mechanics of regular systems should be expected in a more perfect accounting of the symmetry elements of an infinite regular system.

which possesses the space symmetry group $D_{2h}^{(1)}$. In particular, the nature of all the boundary conditions specifying the decoupling (dissociation) of the system of equations under investigation for the mechanical problem is successfully clarified in this paper and specific features of this decoupling are established.

1. A one-dimensional infinite regular mechanical system S, whose symmetry group $D_{2h}^{(1)}$ contains the one-dimensional subgroup G_t of translations with the fundamental vector $\mathbf{a}$ is considered. The elementary cell S_0 is bounded by the planes Π and Π' of the reflections σ and $t_1\sigma$ (t_r denotes a translation by the vector $r\mathbf{a}$). It is easy to see that $\Pi' = t_{0.5}\Pi$.

If $\mathbf{u} = a^{-1}\mathbf{a}$, then the irreducible representations $\boldsymbol{\tau}_\alpha$ $(\alpha = \mathbf{ku})$ of the subgroup G_t are defined by the relationships

$$\boldsymbol{\tau}_\alpha (t_1) = e^{i\alpha} \tag{1.1}$$

where any vector $\mathbf{k}$ from the Brillouin zone is represented as

$$\mathbf{k} = \alpha\mathbf{u}, \quad -\pi \leqslant \alpha \leqslant \pi \tag{1.2}$$

Here $\boldsymbol{\tau}_{\alpha\nu}$ is understood to be the ν-th irreducible representation of dimensionality m_α with the star $\{\alpha\mathbf{u}\}$.

Three kinds of irreducible stars $\{\alpha\mathbf{u}\}$ can be distinguished depending on the absolute value of the scalar α: (a) $\alpha = 0$, (b) $|\alpha| = \pi$, (c) $0 < |\alpha| < \pi$. For the first two kinds, $m_0 = m_\pi = 1$ and $\boldsymbol{\tau}_{\alpha\nu}(\sigma) = (-1)^{\nu-1}$, while $\boldsymbol{\tau}_{0\nu}(t_1) = -\boldsymbol{\tau}_{\pi\nu}(t_1) = 1$. A two-dimensional irreducible representation, whose operator matrix is $\tau_{\alpha 1}(g) = \tau_\alpha(g)$, $\forall g \in D_{2h}^{(1)}$, corresponds to a type (c) star, where the families of the matrix functions $\tau_\alpha(g)$ of the scalar argument α are defined as follows:

$$\tau_\alpha(t_r) = \begin{Vmatrix} \cos r\alpha & \sin r\alpha \\ -\sin r\alpha & \cos r\alpha \end{Vmatrix}, \quad \tau_\alpha(t_r\sigma) = \begin{Vmatrix} \cos r\alpha & -\sin r\alpha \\ -\sin r\alpha & -\cos r\alpha \end{Vmatrix}$$

Let the systems S and S_0 be in the domains Ω and Ω_0, $x(\Pi) \in \Pi \cap \Omega_0$, while $x(\Pi') \in \Pi' \cap \Omega_0$. Because of the orthogonality of the plane Π to the vector $\mathbf{a}$, by constructing a system of coordinate axes from an axis with the direction $\mathbf{u}$ and two orthogonal axes the linear displacement of any point $x \in \Omega$ in the directions of the coordinate axes and the angular displacements of any areas relative to it can be separated into kinds of symmetric p^+ and skew-symmetric p^- factors relative to the plane Π according to the natural criterion

$$\sigma p^\pm(x) = \pm p^\pm(x) \tag{1.3}$$

where $p(x)$ is the value of the factor p at the point $x \in \Omega$.

Further, the m_α of systems S_μ ($\mu = 1, 2, \ldots, m_\alpha$) are investigated, each of which agrees with the system S and whose deformed states are converted by the representation $\boldsymbol{\tau}_{\alpha\nu}$. This latter means that any of the functions p_μ of the mentioned factors of the system S_μ refers to the class of functions $p_{\alpha\nu\mu}$ given in Ω, each of them satisfying the equality

$$p_{\alpha\nu\mu}(gx) = g\sum_{\rho=1}^{m_\alpha} \tau_{\alpha\nu\rho\mu}(g^{-1})\, p_{\alpha\nu\rho}(x), \quad \forall g \in D_{2h}^{(1)} \tag{1.4}$$

where $\tau_{\alpha\nu\rho\mu}$ is the $\rho\mu$-th element of the matrix $\tau_{\alpha\nu}(g)$ of the operator $\boldsymbol{\tau}_{\alpha\nu}(g)$. If it is a question of the forced vibrations of a regular system, then here and henceforth functions of their amplitude values are understood to be the functions of the stress or strain state factors.

The plane $t_r\Pi$ contains the points $t_rx\,(\Pi)$ of the cell t_rS_0 and the points $t_r\sigma x\,(\Pi)$ of the cell $t_r\sigma S_0$. A set C of active relative couplings $C\,(p)$ of the form $p\,[t_rx\,(\Pi)] = p\,[t_r\sigma x\,(\Pi)]$ is superposed on some factors of the strain state at the points $t_rx\,(\Pi)$ and $t_r\sigma x\,(\Pi)$. The set C_1 of relative constraints $C_1\,(p)$ between some factors at the points $t_rx\,(\Pi')$ and $t_{r+1}\sigma x\,(\Pi')$ is introduced analogously. Namely, depending on their type, the following bounds imposed on their values at points belonging to the reflection planes

$$\text{a)}\ \alpha = 0, \begin{cases} \nu = 1;\ p_1^-\,[x\,(\Pi)] = p_1^-\,[x\,(\Pi')] = 0 \\ \nu = 2;\ p_1^+\,[x\,(\Pi)] = p_1^+\,[x\,(\Pi')] = 0 \end{cases} \tag{1.5}$$

$$\text{b)}\ |\,\alpha\,| = \pi, \quad \begin{cases} \nu = 1;\ p_1^-\,[x\,(\Pi)] = p_1^+\,[x\,(\Pi')] = 0 \\ \nu = 2;\ p_1^+\,[x\,(\Pi)] = p_1^-\,[x\,(\Pi')] = 0 \end{cases}$$

$$\text{c)}\ 0 < |\,\alpha\,| < \pi, \quad r = 0, \pm 1, \pm 2, \ldots$$

$$p_2^+\,[x\,(t_r\Pi)] = -p_1^+\,[x\,(t_r\Pi)]\ \mathrm{tg}\ r\alpha$$

$$p_2^-\,[x\,(t_r\Pi)] = p_1^-\,[x\,(t_r\Pi)]\ \mathrm{ctg}\ r\alpha$$

$$p_2^+[x\,(t_r\Pi')] = -p_1^+\,[x\,(t_r\Pi')]\ \mathrm{tg}\,\frac{2r+1}{2}\,\alpha$$

$$p_2^-\,[x\,(t_r\Pi')] = p_1^-\,[x\,(t_r\Pi')]\ \mathrm{ctg}\,\frac{2r+1}{2}\,\alpha$$

are found successfully from (1.4) for these factors depending on their type.

With respect to the set of cells $S_\mu{}^{(0)}$ ($\mu = 1, 2, \ldots, m_\alpha$), each of which agrees with the cell t_rS_0, it is natural to treat any condition for the factor p from (1.5) as some ideal mechanical constraint $C_{\alpha\nu}{}^{(r)}\,(p)$ (in the $t_r\Pi$ plane) or $C_{\alpha\nu 1}^{(r)}\,(p)$ (in the $t_r\Pi'$ plane), corresponding to the $\alpha\nu$-th irreducible representation of the group $D_{2h}{}^{(1)}$ for fixed r. The lack of constraints for one of the kinds of strain state factors in (1.5) should be considered as passivity of similar constraints. Since a mutually one-to-one coupling $C_{\alpha\nu}{}^{(r)}\,(p)$ corresponds to each constraint $C\,(p)$ from the set C (an analogous assertion is valid for the sets C_1 and $C_{\alpha\nu 1}^{(r)}$), then passive couplings can also enter into the sets $C_{\alpha\nu}{}^{(r)}$ and $C_{\alpha\nu 1}{}^{(r)}$ in addition to the appropriate subsets $C_{\alpha\nu}{}^{(r)}$ and $c_{\alpha\nu 1}$ of active couplings. A mechanical system consisting of m_α cells $S_\mu{}^{(0)}$ ($\mu = 1, 2, \ldots, m_\alpha$), on which the sets of couplings $C_{\alpha\nu}{}^{(r)}$ and $C_{\alpha\nu 1}{}^{(r)}$ have been imposed is called the r-th elementary cell in the sense of the $\alpha\nu$-th representation and is denoted by $S_{\alpha\nu}{}^{(r)}$. It can be shown that the following theorem holds.

Theorem 1. If a load $q_{\alpha\nu\mu}$ acts on a system S, then the strain and stress state of its cell t_rS_0 agrees with the corresponding state of the cell $S_\mu{}^{(0)}$ of the elementary system $S_{\alpha\nu}{}^{(r)}$ for which the load $q_\rho{}^{(0)}$ of the cell $S_\rho{}^{(0)}$ is determined by the relationship

$$q_\rho^{(0)}\,(x) = q_{\alpha\nu\rho}\,(x), \quad \forall x \in t_r\Omega_0, \quad \rho = 1, 2, \ldots, m_\alpha \tag{1.6}$$

2. If a finite mechanical system is regular, then under some boundary conditions a method to represent it as elementary can exist for which the system S will possess the symmetry group $D_{2h}^{(1)*}$ with the fundamental vector $\mathbf{a}^* = \mathbf{a}\,/\,n$, where n is the number of elementary cells relative to the group $D_{2h}^{(1)*}$ which generate the initial finite system. An asterisk is henceforth used to denote a number of concepts associated with the group $D_{2R}^{(1)*}$.

Let C^*, C_1^*, K^*, K_1^* be sets of active rigid constraints imposed on the relative (C^* and C_1^*) or absolute (K^* and K_1^*) displacements and rotations of adjacent cells of a regular system at any of its internal sections cut by the planes $t_r^*\Pi$ and $t_{-r}^*\Pi'$ ($r = 1, 2, \ldots$); and let K' and K_1' be sets of the same constraints imposed on its absolute displacements and rotations at the sections Π and Π' (t_r^* is understood to be the translation by a vector $r\mathbf{a}^*$). If the system S possesses the symmetry group $D_{2h}^{(1)*}$, then $C = C^*$ and $C_1 = C_1^*$. The following assertion then results from the above: a finite regular system is elementary relative to an infinite system S with symmetry group $D_{2h}^{(1)*}$ in the sense of the $\alpha\nu$-th irreducible representation of the group $D_{2h}^{(1)}$ if $K' = K^* \bigcup c_{\alpha\nu}^{(r)}$ and $K_1' = K_1^* \bigcup c_{\alpha\nu 1}^{(r)}$, where $C_{\alpha\nu}^{(r)}$ and $C_{\alpha\nu 1}^{(r)}$ should correspond to the sets C^* and C_1^*.

The value of the criterion introduced is that regular one-dimensional systems satisfying it admit of the above-mentioned simplifications in the computations. In fact, according to Theorem 1, instead of such systems considered as elementary in the sense of the $\alpha\nu$-th irreducible representation of the group $D_{2h}^{(1)}$, it is possible to investigate m_α of systems S_μ whose load functions $q_{\alpha\nu\mu}$ are determined by (1.6) and (1.4) rewritten as

$$gp_{\alpha\nu\mu} = \sum_{\rho=1}^{m_\alpha} \tau_{\alpha\nu\rho\mu}(g)\, p_{\alpha\nu\rho}, \quad \forall g \in D_{2h}^{(1)} \tag{2.1}$$

by using here the higher density of the symmetry group $D_{2h}^{(1)*}$ relative to $D_{2h}^{(1)}$.

If ψ is understood to be the function in the group which yields to averaging by using the averaging functional introduced, then this functional can be determined by the usual method for the groups $D_{2h}^{(1)}$ and G_t [5]:

$$M_D(\psi) = \frac{1}{2} \lim_{\eta\to\infty} \frac{1}{2\eta+1} \sum_{r=-\eta}^{\eta} [\psi(t_r) + \psi(\sigma t_r)] \tag{2.2}$$

$$M_t(\psi) = \lim_{\eta\to\infty} \frac{1}{2\eta+1} \sum_{r=-\eta}^{\eta} \psi(t_r)$$

The linear space L extended to the functions $t_r^* p_{\alpha\nu\mu}$ ($\mu = 1, 2, \ldots, m_\alpha$; $r = 0, 1, \ldots, n-1$) is invariant relative to all elements of the group $D_{2h}^{(1)*}$ on the basis of (2.1). The representation $\mathbf{T}$ of this group for which

$$T(g^*)p = g^*p, \quad \forall g^* \in D_{2h}^{(1)*}, \quad \forall p \in L \tag{2.3}$$

operates in L.

The operators $\mathbf{T}(t_r^*)$ ($r = 0, \pm 1, \pm 2, \ldots$) form the representation $\mathbf{T}_t$ of the subgroup G_t in the space L.

Let (p, f) be some scalar product of the functions p and f from the space L, which is bounded for the functions $t_r p_{\alpha\nu\mu}$ ($\mu = 1, 2, \ldots, m_\alpha$; $r = 0, 1, 2, \ldots, n-1$). Let us introduce a new scalar product

$$\{p, f\} = M_D^* [(g^*p, g^*f)] \tag{2.4}$$

in the space L for which the representations $\mathbf{T}$ and $\mathbf{T}_t$ are unitary, and therefore, decompose into irreducible representations of these groups. Because of the boundedness of the matrix elements of the representations $\mathbf{T}$, $\mathbf{T}_t$ and the irreducible representations of the groups considered, the averaging functionals introduced can be applied to these elements to establish known orthogonality properties [5].

The representations $\mathbf{T}$ and $\tau_{\alpha\nu}$ are interrelated as follows:

$$\mathbf{T}(g)\,p_{\alpha\nu\mu} = \tau_{\alpha\nu}(g)\,p_{\alpha\nu\mu}, \quad \forall g \in D_{2h}^{(1)}, \quad \mu = 1, 2, \ldots, m_\alpha \tag{2.5}$$

There exists a linear combination of p_α functions $p_{\alpha\nu\mu}$ ($\mu = 1, 2, \ldots, m_\alpha$) which, subjected to the translation t_r ($r = 0, \pm 1, \pm 2, \ldots$), is converted in conformity with the vector $\alpha\mathbf{u}$. The space $L_{t\alpha} \subset L$ generated by the functions $t_r{}^* p_\alpha$ ($r = 0, 1, \ldots, n-1$), is invariant relative to the representation $\mathbf{T}_t$ on the basis of (1.1), (2.3) and (2.5), inducing a representation $\mathbf{T}_{t\alpha}$ with the character $\chi_{t\alpha}$ in $L_{t\alpha}$. If the functions $p_{\alpha\nu\mu}$ ($\mu = 1, 2, \ldots, m_\alpha$) do not possess any special properties, then the functions $t_r{}^* p_\alpha$ ($r = 0, 1, \ldots, n-1$) are linearly independent and

$$\chi_{t\alpha}(t_r{}^*) = \delta_{r,\,s\cdot n}\, n e^{is\alpha}$$

where $\delta_{r,s\cdot n}$ is the Kronecker delta and s is an arbitrary integer. The number of times which the irreducible representation τ_ε of the subgroup $G_t{}^*$ is encountered in the representation $\mathbf{T}_{t\alpha}$ is

$$m_\varepsilon = M_t^*\,[\chi_{t\alpha}(t_r{}^*)e^{-ir\varepsilon}] = \delta_{\varepsilon\beta} \tag{2.6}$$

where

$$\beta = n^{-1}(\alpha + j2\pi), \quad j = 0, \pm 1, \pm 2, \ldots \tag{2.7}$$

Let K_α be the set of values of β which differ in absolute value, satisfy the inequality (1.2) and are determined from (2.7). Then the set $K_{-\alpha}$ consists of numbers of the form $n^{-1}(-\alpha - 2\pi j)$ and contains the number β if and only if $m_\alpha = 1$. Indeed, if $\alpha + 2\pi j_1 = -\alpha - 2\pi j_2$, then $\alpha = -(j_1 + j_2)\pi$. Consequently, since $L = L_{t\alpha} \cup L_{t(-\alpha)}$, the set K_α determines completely the irreducible stars $\{\beta\mathbf{u}\}$ which form a star of representations T. Moreover, there results from (2.6) that the representation $\tau_{\beta\rho}^*$ of the group $D_{2h}^{(1)*}$ induced by the representation $\mathbf{T}$ in a linear shell L_β of the functions $p_\beta{}^*$ and $\sigma p_\beta{}^*$ converted under the effect of the translations t_r ($r = 0, \pm 1, \pm 2, \ldots$) to the vectors $\beta\mathbf{u}$ and $(-\beta\mathbf{u})$, respectively, is irreducible. Meanwhile

$$p_\beta{}^* = \sum_{r=0}^{n-1} e^{-ir\beta} t_r{}^* p_\alpha, \quad \forall \beta \in K_\alpha \tag{2.8}$$

since in connection with (2.3), (2.5) and (2.7)

$$t_1{}^* p_\beta{}^* = \sum_{r=1}^{n-1} e^{i\beta} e^{-ir\beta} t_r{}^* p_\alpha + e^{-i(n-1)\beta}\tau_{\alpha\nu}(t_1)\,p_\alpha = e^{i\beta} p_\beta{}^*$$

and by analogy

$$\sigma q_\beta^* = \sum_{r=0}^{n-1} e^{ir\beta} t_r{}^* \sigma p_\alpha, \quad \forall \beta \in K_\alpha \tag{2.9}$$

In case the star $\{\beta\mathbf{u}\}$, and therefore, the star $\{\alpha\mathbf{u}\}$ also refer to the types (a) or (b), then (2.9) is rewritten as

$$\tau_{\beta\rho}{}^*(\sigma)\,p_\beta{}^* = \tau_{\alpha\nu}(\sigma) \sum_{r=0}^{n-1} e^{-r\beta i} t_r{}^* p_\alpha$$

from which there follows that $\tau_{\beta\rho}{}^*(\sigma) = \tau_{\alpha\nu}(\sigma)$.

The following theorem has thereby been proved.

T h e o r e m 2. The space L generated by the functions $t_r^* p_{\alpha\nu\mu}$ ($\mu = 1, 2, \ldots, m_\alpha$; $r = 0, 1, \ldots, n-1$) decomposes into a subspace L_β, orthogonal in the sense of the scalar product (2.4) and converted in conformity with the irreducible representations

$\tau_{\beta 1}{}^*$ if $0 < |\beta| < \pi$ and with the representations $\tau_{\beta\nu}{}^*$ in other cases, where the real numbers β differ in absolute value and are determined from (2.7) and (1.1).

3. If we put $p^*_{\beta\eta} = p^*_{\beta 1\eta}$ for $0 < |\beta| < \pi$ and $p_{\beta\eta}{}^* = \delta_{\eta\rho}\, p^*_{\beta\rho_1}$ for $\beta = 0$ or $\beta = |\pi|$ in the subspace L_β, convertible according to the representation $\tau_{\beta\rho}{}^*$, then those of the functions $p_{\beta\eta}{}^*$ which are non-zero will form a basis therein. In the case of two-dimensionality of the representation $\tau_{\beta\rho}{}^*$ for any $g^* \in D_{2h}{}^{(1)*}$

$$\begin{aligned} &\mathbf{T}(g)^*\, P^*_{\beta\varphi} = \sum_{\eta=1}^{2} \tau^*_{\beta\eta\varphi}(g^*)\, p^*_{\beta\eta}, \quad \varphi = 1, 2 \\ &\mathbf{T}(g^*) p_\beta{}^* = \tau_{\beta 11}{}'(g^*) p_\beta{}^* + \tau_{\beta 21}{}'(g^*)\sigma p_\beta{}^*, \quad \forall\beta \in K_\alpha \\ &\mathbf{T}(g^*)\sigma p_\beta{}^* = \tau_{\beta 12}{}'(g^*) p_\beta{}^* + \tau_{\beta 22}{}'(g^*)\sigma p_\beta{}^* \end{aligned} \tag{3.1}$$

Here

$$\tau_\beta{}'(t_r{}^*) = \begin{Vmatrix} e^{ir\beta} & 0 \\ 0 & e^{-ir\beta} \end{Vmatrix}, \quad \tau_\beta{}'(\sigma) = \begin{Vmatrix} 0; & 1 \\ 1; & 0 \end{Vmatrix}$$

$\tau^*_{\beta\eta\varphi}(g^*)$ and $\tau'_{\beta\eta\varphi}(g^*)$ are $\eta\varphi$-th elements of the matrices $\tau_\beta{}^*(g^*)$ and $\tau_\beta{}'(g^*)$. The last matrices are similar (A is the similarity conversion matrix) and therefore $A p_\beta{}^* = p_{\beta 1}{}^*$ and $A\sigma p_\beta{}^* = p_{\beta 2}{}^*$.

Let $a_{\varphi\eta}$ and $b_{\varphi\eta}$ ($\varphi, \eta = 1, 2$) denote furthermore the $\varphi\eta$-th elements of the matrices A and A^{-1}.

From the above it follows

$$p^*_{\beta\eta} = a_{1\eta} p_\beta{}^* + a_{2\eta}\sigma p_\beta{}^*, \quad p^*_\beta = \sum_{\eta=1}^{2} b_{\eta 1} p^*_{\beta\eta}, \quad \sigma p_\beta{}^* = \sum_{\eta=1}^{2} b_{\eta 2} p^*_{\beta\eta} \tag{3.2}$$

The validity of (3.1) and (3.2) can be confirmed directly for one-dimensional representations and it can be established by using (2.7) that

$$\tau_\alpha(g) = \tau_\beta{}^*(g) \quad (0 \leqslant |\alpha| \leqslant \pi), \quad \forall g \in D_{2h}^{(1)}, \quad \forall\beta \in K_\alpha \tag{3.3}$$

By using the formulas (3.1) – (3.3), (2.8), (2.9), (2.2), (2.4) and the evident unitarity of the matrix A this affords the possibility of deducing the following important relationships:

$$\begin{aligned} p^*_{\beta\eta} &= a_{1\eta} \sum_{r=0}^{n-1} e^{-ir\beta} t_r{}^* \sum_{\varphi=1}^{2} b_{\varphi 1} p_{\alpha\varphi} + a_{2\eta} \sum_{r=0}^{n-} e^{ir\beta} t_r{}^* \sum_{\varphi=1}^{2} b_{\varphi 2} p_{\alpha\varphi} = \\ &\sum_{\varphi=1}^{2} \sum_{r=0}^{n-1} \tau^*_{\beta\eta\varphi}(t_r{}^*)\, t_r{}^* p_{\alpha\varphi} \quad (\eta = 1, 2), \quad \forall\beta \in K_\alpha \end{aligned} \tag{3.4}$$

$$\begin{aligned} \{p^*_{\beta\eta}, p^*_{\beta\nu}\} &= \sum_{\rho=1}^{2} \sum_{\varphi=1}^{2} (p^*_{\beta\rho}, p^*_{\beta\varphi})\, M_D{}^*[\tau^*_{\beta\rho\eta}(g^*)\, \tau^*_{\beta\varphi\nu}(g^*)] = \\ &\frac{\delta_{\nu\eta}}{m_\beta} \sum_{\rho=1}^{2} (p^*_{\beta\rho}, p^*_{\beta\rho}) \quad (\eta = 1, 2), \quad \forall\beta \in K_\alpha \end{aligned} \tag{3.5}$$

$$\begin{aligned} \{p^*_{\beta\eta}, p_{\alpha\nu}\} &= \sum_{\rho=1}^{2} \sum_{\varphi=1}^{2} \sum_{r=1}^{n-1} (t_r{}^* p_{\beta\rho}, t_r{}^* p_{\alpha\varphi}) \frac{1}{n} M_D\, [\tau_{\alpha\rho\eta}(g)\, \tau_{\alpha\varphi\nu}(g)] = \\ &\frac{\delta_{\eta\nu}}{m_\alpha n} \sum_{\varphi=1}^{2} \sum_{r=0}^{n-1} (t_r{}^* p^*_{\beta\varphi}, t_r{}^* p_{\alpha\varphi}) \quad (\eta = 1, 2), \quad \forall\beta \in K_\alpha \end{aligned} \tag{3.6}$$

since $M_D[\tau_{\alpha\rho\eta}(g)\tau_{\alpha\varphi\nu}(g)] = \delta_{\rho\varphi}\delta_{\nu\eta}\ {}^1/_2$ for $0 < |\alpha| < \pi$, while $M_D[\tau_{\alpha\rho\eta}(g)\tau_{\alpha\varphi\nu}(g)] = \delta_{\rho\varphi}\delta_{\eta\nu}\delta_{\varphi\nu}$, for $|\alpha| = \pi$ or $\alpha = 0$ but $p_{\alpha\varphi}{}^* = \delta_{\varphi\nu}p_{\alpha\nu}{}^*$.

The expression (3.6) is rewritten as

$$\{p_{\beta\eta}^*, p_{\alpha\nu}\} = \frac{\delta_{\eta\nu}}{m_\alpha n}\sum_{\varphi=1}^{2}\sum_{r=0}^{n-1}\Big(\sum_{\rho=1}^{2}\tau_{\beta\rho\varphi}^*(t_r{}^*)\,p_{\beta\rho}^*,\ t_r p_{\alpha\varphi}\Big) = \tag{3.7}$$

$$\frac{\delta_{\eta\nu}}{m_\alpha n}\sum_{\rho=1}^{2}(p_{\beta\rho}^*,\ p_{\beta\rho}^*) \quad (\eta = 1, 2), \quad \forall\beta \in K_\alpha$$

The equality (3.7) means that the system of non-zero functions $p_{\beta\eta}{}^*$ from all the subspaces $L_\beta \subset L$ forms an orthogonal basis of the space L in the sense of the scalar product (2.5). According to (3.5) and (3.7)

$$\frac{\{p_{\beta\eta}^*, p_{\alpha\nu}\}}{\{p_{\beta\eta}^*, p_{\beta\eta}^*\}} = \delta_{\eta\nu}\frac{m_\beta}{m_\alpha n} \quad (\eta = 1, 2) \quad \forall\beta \in K_\alpha$$

In this connection, the fact that $p_{\alpha\nu} \in L$ makes the following assertion evident.

Theorem 3. The functions $p_{\alpha\nu}$ are representable as

$$p_{\alpha\nu} = \sum_{\beta\in K_\alpha}\frac{m_\beta}{m_\alpha n}p_{\beta\nu}^*$$

If m_K is the number of elements in the set K_α, then Theorems 1 and 2, the superposition principle at the basis of Theorem 3, as well as (2.7) and (3.4) permit the investigation of m_K uniquely defined null elementary systems $S_{\beta\rho}{}^{(0)}$ (in the sense of representations of the group $D_{2h}{}^{(1)*}$) in place of $S_{\alpha\nu}{}^{(r)}$. The results obtained by using (1.4) can be extended to the linear system $S_{\alpha\nu}{}^{(r)}$. This indicates the mechanical meaning of the simplifications inserted in the computation of regular systems for a number of boundary conditions.

In conclusion, two circumstances should be noted. Firstly, all the above remains true even for loads for which the dimensionality of the space $L_{t\alpha}$ is less than n. In this case, according to (3.4), the load functions of the systems $S_{\beta\rho}^{(0)*}$ turn out to be identically zero for some values of β. Secondly, the theorems presented are applicable for finding the natural frequency spectrum or the critical forces. As is easy to see, the spectrum of the elementary system $S_{\alpha\nu}{}^{(r)}$ is a combination of appropriate spectra of the systems $S_{\beta\rho}^{(0)*}$.

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